Bull. Math. Soc. Sci. Math. Roumanie
Tome 51(99) No. 3, 2008, 245–252

Extensions of discrete finite rank valued fields *

by
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Abstract

We provide some sufficient conditions for a valued subfield (K', v') of a valued field (K, v) to be of finite codimension in K, i.e. $[K : K']$ be finite. Moreover, these conditions on K' imply the fundamental equality $ef = [K : K']$ and the unicity of the valuation v as an extension of v'.

Key Words: Valued fields, general valuations, the fundamental equality, complete fields.
2000 Mathematics Subject Classification: Primary 12J10,12J20, Secondary 13A18.

Introduction

A basic result in the classical valuation theory says that if $(K', v') \subset (K, v)$ is a finite extension of valued fields, then the ramification index e and the inertia degree f are finite and $ef \leq n = [K : K']$ (see [R]). If (K, v) is a discrete rank 1 valued field and if (K', v') is complete, then $ef = n$ and (K, v) is also complete ([Neu]). In this paper we consider the reverse problem: "In what conditions on (K', v'), can we state that $[K : K'] < \infty$?". Namely, we prove that if $(K', v') \subset (K, v)$ is an extension of rank t discrete valued fields such that (K', v') is strongly complete (see Definition 1.1) and $e, f < \infty$, then $[K : K'] = ef$ and K is also strongly complete (Theorem 3.1). We also prove that if $[K : K'] = n < \infty$ and if (K', v') is a strongly complete discrete rank t valued subfield of (K, v), then v is the unique extension of v' to K and $ef = n$ (Corollary 3.1). If $t = 1$, strongly complete means the usual notion of a topological completion of a rank 1 valued field. Theorem 2.1 is a kind of "inverse" of [Se], Prop. 3, §2, Ch. II. Theorem 3.1 extends Theorem 2.1 to the case of an arbitrary finite rank.

*This paper was partially supported by the Higher Education Commission of Pakistan.

1 Definitions and notations

Let (K, v) be a (Krull) valued field and let $(G_v, +, \leq)$ be its value group. A rank t discrete valued field (K, v) is a valued field (K, v) with its value group $(G_v, +, \leq)$ of rank t, which is isomorphic to $\mathbb{Z} \times \mathbb{Z} \times ... \times \mathbb{Z}$, t-times.

Let K' be a subfield of the valued field (K, v) and let v' be the restriction of v to K'. We say that $(K', v') \subset (K, v)$ is an extension of valued fields. We also say that (K', v') is a valued subfield of (K, v). Let $\mathcal{O}_v = \{x \in K \mid v(x) \geq 0\}$ be the valuation ring of (K, v), let $\mathfrak{m}_v = \{x \in \mathcal{O}_v \mid v(x) > 0\}$ be the maximal ideal of (K, v) and let $\overline{K}_v = \mathcal{O}_v/\mathfrak{m}_v$ be the residue field of (K, v). Let $G_{v'}, \mathcal{O}_{v'}, \mathfrak{m}_{v'}$ and $\overline{K'}_{v'}$ be the value group, the valuation ring, the maximal ideal and the residue field respectively for the valued subfield (K', v') of (K, v).Let $e = e(K/K') = [G_v : G_{v'}]$ be the ramification index of the valued field extension $(K', v') \subset (K, v)$ and let $f = f(K/K') = [\overline{K}_v : \overline{K'}_{v'}]$ be its inertia degree.

Let (K, v) be a discrete rank t $(t > 1)$ valued field with the value group $G_v = \mathbb{Z} \times \mathbb{Z} \times \cdots \times \mathbb{Z} = \mathbb{Z}^t$, lexicographically ordered. For any $x \in K$, let us denote $v(x) = (v(x)_1, v(x)_2, \ldots, v(x)_t) \in \mathbb{Z}^t$. If we denote $w_1(x) = v(x)_1$, we get that $w_1 : K \to \mathbb{Z} \cup \{\infty\}$ is a discrete rank 1 valuation, called the *marginal valuation* of v. The valuation ring $\mathcal{O}_{w_1}$ of w_1 contains the valuation ring $\mathcal{O}_v$ of v. For $i = 1, 2, \ldots, t$, let us choose π_i in $\mathcal{O}_v$ such that $v(\pi_i) = (0, \ldots, 0, 1, 0, \ldots, 0) \in \mathbb{Z}^t$, where 1 is in the i-th position; π_i is called an *i-th uniformizer* of (K, v). Hence $\mathfrak{m}_{w_1} = \pi_1 \mathcal{O}_{w_1}$, the ideal generated by π_1 in $\mathcal{O}_{w_1}$. The residue field $\overline{K}_{w_1} = \mathcal{O}_{w_1}/\pi_1\mathcal{O}_{w_1}$ has a natural valuation v_1 of rank $t - 1$ on it. Its valuation ring corresponds to the image of the valuation ring $\mathcal{O}_v$ through the canonical surjective map: $\mathcal{O}_{w_1} \to \mathcal{O}_{w_1}/\pi_1\mathcal{O}_{w_1}$. Thus one can set $v_1(\bar{x}) = (v(x)_2, \ldots, v(x)_t) \in \mathbb{Z}^{t-1}$. Now we consider the discrete rank $t - 1$ valued field $(\overline{K}_{w_1}, v_1)$. The residue field of $(\overline{K}_{w_1}, v_1)$ is also $\overline{K}_v = \mathcal{O}_v/\mathfrak{m}_v$, the residue field of (K, v).

Let now $(K', v') \subset (K, v)$ be a valued subfield of (K, v). One makes the same construction as above for (K', v') and obtains: $\mathcal{O}_{v'} \subset \mathcal{O}_v$, $\mathfrak{m}_{v'} \subset \mathfrak{m}_v$, $\overline{K'}_{v'} \subset \overline{K}_v$, $G_{v'} \subset G_v$, $w'_1 : K' \to \mathbb{Z} \cup \{\infty\}$, the rank 1 marginal valuation of $v' : w'_1(x) = v'(x)_1$, $\overline{K'}_{w'_1} \subset \overline{K}_{w_1}$ and $v'_1 : \overline{K'}_{w'_1} \to \mathbb{Z}^{t-1} \cup \{\infty\}$. The residue field of $(\overline{K'}_{w'_1}, v'_1)$ is $\overline{K'}_{v'}$, the residue field of (K', v').

If we start with the valued field extension $(\overline{K'}_{w'_1}, v'_1) \subset (\overline{K}_{w_1}, v_1)$ and do the same as above we obtain a new valued field extension $(\overline{K'}_{w'_2}, v'_2) \subset (\overline{K}_{w_2}, v_2)$ such that $(\overline{K}_{w_2}, v_2)$ has rank $t - 2$ and $v'_2(\bar{x}) = (v(x)_3, \ldots, v(x)_t) \in \mathbb{Z}^{t-2}$ for $\overline{x} \neq 0$, and so on. We call the rank 1 valuations $w_1, w_2, \ldots, w_t$, *the principal valuations* of (K, v) and the valuations $v_1, v_2, \ldots, v_{t-1}$ of ranks $t-1, t-2, \ldots, 1$ respectively, *the auxiliary valuations* of (K, v). The valuation w_i is said to be *the i-th principal valuation* of (K, v). It is clear that w_i is the marginal valuation of v_{i-1}, where $i = 1, 2, \ldots, t$. Here $v_0 = v$.

Let (K, v) be a valued field and let ∞ be the biggest element of $G \cup \{\infty\}$. A sequence $\{x_n\}$, $x_n \in K$ is called a *Cauchy sequence* if the set $\{v(x_{n+1} - x_n)\}$ is not bounded in G. A sequence $\{x_n\}$ in K is *convergent to* $x \in K$ if the set $\{v(x_n - x)\}$ is not bounded in G. This convergence endows K with a structure of a topological field. One calls this last topology the v-adic topology on K. We say that (K, v) is *complete* if any Cauchy sequence in K is convergent to a unique element x of K.

Definition 1.1. *Let (K, v) be a discrete rank t valued field and let $v_0 = v, v_1, ..., v_{t-1}$ be the auxiliary valuations of (K, v) defined on the field K, $\overline{K}_{w_1}$, $\overline{K}_{w_2}$, ..., $\overline{K}_{w_{t-1}}$ respectively. If the valued fields $(K, v), (\overline{K}_{w_1}, v_1), ..., (\overline{K}_{w_{t-1}}, v_{t-1})$ are complete, one says that (K, v) is strongly complete.*

Example 1.1. *Let k be a field and let X_1, X_2, X_3 be three independent variables over k. Let $k((X_3))$ be the field of Laurent power series $s = \sum_{i \geq i_0} a_i X_3^i$, $i_0 \in \mathbb{Z}$, $a_{i_0} \neq 0, a_i \in k$. Let $k((X_3))((X_2))$ be the field of Laurent power series $q = \sum_{j \geq j_0} B_j X_2^j$, $j_0 \in \mathbb{Z}$, $B_{j_0} \neq 0$, $B_j \in k((X_3))$. Let $K = k((X_3))((X_2))((X_1))$ be the field of Laurent power series $r = \sum_{t \geq t_0} C_t X_1^t$, $C_{t_0} \neq 0$, $C_t \in k((X_3))((X_2))$. It is clear now that any element r of K can be uniquely written as*

$$r = \sum_{(t,j,i) \succeq (t_0, j_0, i_0)} a_{ijt} X_1^t X_2^j X_3^i,$$

where the monomials $a_{ijt} X_1^t X_2^j X_3^i$ are lexicographically ordered like the triplets (t, j, i) in $\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$. Now we define: $v(r) = (t_0, j_0, i_0)$, $w_1(r) = t_0$, $v_1(q) = (j_0, t_0)$, $w_2(q) = j_0$, $v_2(s) = i_0 = w_3(s)$. It is easy to see that (K, v) is strongly complete (see Definition 1.1) but $k((X_3))(X_2)((X_1))$ is not strongly complete because $(\overline{K}_{w_1}, v_1)$ is not complete. If instead of X_1, X_2, X_3 we consider X_1, $X_2, ..., X_t, t$ independent variables over k, we get a discrete valued field $K = k((X_t))((X_{t-1}))...((X_1))$ of rank t, which is strongly complete. Following [Se] for instance, one can generalize the structure theorems for local fields to these discrete rank t strongly complete fields. In the "equal characteristic case" the only discrete rank t strongly complete valued fields (K, v) (up to isomorphisms of valued fields) are of the form $K = k((X_t))((X_{t-1}))...((X_1))$.

2 The rank 1 case

Theorem 2.1. *Let $(K', v') \subset (K, v)$ be a discrete rank 1 valued field extension with (K', v') complete. We assume that the ramification index $e = e(K/K')$ and the inertia degree $f = f(K/K')$ are finite. Then, the valuation ring $\mathcal{O}_v$ is a free $\mathcal{O}_{v'}$-module of rank ef and $[K : K']$ is finite and equal to ef. Moreover, (K, v) is also complete and v is the unique valuation which extend v' to K.*

Proof: Let π_1 and π'_1 be uniformizers of $\mathcal{O}_v$ and of $\mathcal{O}_{v'}$ respectively i.e. $v(\pi_1) = 1$ and $v'(\pi'_1) = e$. Thus we can write $\pi'_1 = \pi_1^e u$, where u is a unit in $\mathcal{O}_v$. Let $\mathfrak{m}_v = \pi_1 \mathcal{O}_v$ be the maximal ideal of $\mathcal{O}_v$.

Let $A \in \mathcal{O}_v$ and let us denote by $\bar{A}$ the class of A in $\overline{K}_v$. Since $\overline{K}_v$ is finite over $\overline{K'}_{v'}$, let $\{\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_f\}$ be a basis of $\overline{K}_v$ over $\overline{K'}_{v'}$ $(v_1, v_2, \ldots, v_f \in \mathcal{O}_v)$, then:

$$A = \sum_{i=1}^{f} a_{i,0}^{(0)} v_i + \pi_1 A_1,\ A_1 \in \mathcal{O}_v \text{ and } a_{i,0}^{(0)} \in \mathcal{O}_{v'}.$$

Applying the same procedure to $A_1 \in \mathcal{O}_v$, we get:

$$A = \sum_{i=1}^{f} a_{i,0}^{(0)} v_i + \sum_{i=1}^{f} a_{i,1}^{(0)} v_i \pi_1 + \pi_1^2 A_2,$$

where $A_2 \in \mathcal{O}_v$ and $a_{i,1}^{(0)} \in \mathcal{O}_{v'}$. After e steps one obtains:

$$A = \sum_{j=0}^{e-1} (\sum_{i=1}^{f} a_{i,j}^{(0)} v_i) \pi_1^j + \pi_1^e A_e,\ A_e \in \mathcal{O}_v,\ a_{i,j}^{(0)} \in \mathcal{O}_{v'}.$$

But $\pi_1' = \pi_1^e u$, where u is a unit in $\mathcal{O}_v$. Then:

$$A = \sum_{j=0}^{e-1} (\sum_{i=1}^{f} a_{i,j}^{(0)} v_i) \pi_1^j + \pi_1' B_1,\ B_1 = u^{-1} A_e \in \mathcal{O}_v.$$

Applying the same construction to B_1, one gets:

$$A = \sum_{j=0}^{e-1} \sum_{i=1}^{f} a_{i,j}^{(0)} v_i \pi_1^j + \pi_1' \sum_{j=0}^{e-1} \sum_{i=1}^{f} a_{i,j}^{(1)} v_i \pi_1^j + (\pi_1')^2 B_2,$$

where $B_2 \in \mathcal{O}_v$ and $a_{i,j}^{(1)} \in \mathcal{O}_{v'}$. We continue in this way and, after n steps, we get:

$$A = \sum_{j=0}^{e-1} \sum_{i=1}^{f} (\sum_{k=0}^{n-1} a_{i,j}^{(k)} (\pi_1')^k) v_i \pi_1^j + (\pi_1')^n B_n,\ B_n \in \mathcal{O}_v$$

and $a_{i,j}^{(k)} \in \mathcal{O}_{v'}$ for all $i = 1, 2, \ldots, f,\ j = 0, 1, \ldots, e-1$, and $k = 0, 1, \ldots, n-1$. Since $\mathcal{O}_{v'}$ is closed in K', the convergent series $c_{ij} = \sum_{k=0}^{\infty} a_{i,j}^{(k)} (\pi_1')^k$ are elements in $\mathcal{O}_{v'}$. Moreover, $(\pi_1')^n B_n \to 0$ in $\mathcal{O}_v$, when $n \to \infty$, so

$$A = \sum_{j=0}^{e-1} \sum_{i=1}^{f} c_{ij} v_i \pi_1^j,\ c_{ij} \in \mathcal{O}_{v'}. \tag{1}$$

This means that $\{v_i \pi_1^j,\ i = 1, 2, \ldots, f\ ,\ j = 0, 1, \ldots, e-1\}$ is a generating system of the module $\mathcal{O}_v$ over $\mathcal{O}_{v'}$. We now prove that the expression 1 is unique. For this, it is sufficient to prove that

$$\sum_{j=0}^{e-1} \sum_{i=1}^{f} c_{ij} v_i \pi_1^j = 0,\ c_{ij} \in \mathcal{O}_{v'} \tag{2}$$

implies $c_{ij} = 0$ for $i = 1, 2, \ldots, f$ and $j = 0, 1, \ldots, e-1$. Taking classes in $\overline{K}_v = O_v/\mathfrak{m}_v$, where $\mathfrak{m}_v = \pi_1 \mathcal{O}_v$, we obtain:

$$\sum_{i=1}^{f} \bar{c}_{i0} \bar{v}_i = 0.$$

Since $\{\bar{v}_i\}$, $i = 1, 2, \ldots, f$ is a basis of $\overline{K}_v$ over $\overline{K'}_{v'}$, we get $c_{i0} = \pi_1' c_{i0}'$ where $c_{i0}' \in \mathcal{O}_{v'}$. Now assume that we have proved that $c_{ij} = \pi_1' c_{ij}'$, $c_{ij}' \in \mathcal{O}_{v'}$ for $j = 0, 1, \ldots, r-1$ and $i = 1, 2, \ldots, f$, where $0 < r < e$. Then 2 implies that

$$\pi_1' \sum_{j=0}^{r-1} \sum_{i=1}^{f} c_{ij}' v_i \pi_1^j + \sum_{j=r}^{e-1} \sum_{i=1}^{f} c_{ij} v_i \pi_1^j = 0,\ c_{ij}', c_{ij} \in \mathcal{O}_{v'}.$$

Let us put $\pi_1' = \pi_1^e u$, where u is a unit in $\mathcal{O}_v$ and then simplify π_1^r. The last equation becomes:

$$\pi_1^{e-r} u \sum_{j=0}^{r-1} \sum_{i=1}^{f} c_{ij}' v_i \pi_1^j + \sum_{j=r}^{e-1} \sum_{i=1}^{f} c_{ij} v_i \pi_1^{j-r} = 0,\ c_{ij}',\ c_{ij} \in \mathcal{O}_{v'}.$$

Taking classes in $\overline{K}_v$, we obtain:

$$\sum_{i=1}^{f} \bar{c}_{ir} \bar{v}_i = 0.$$

Since $\{\bar{v}_i\}$, $i = 1, 2, \ldots, f$ is a basis of $\overline{K}_v$ over $\overline{K'}_{v'}$ we get $c_{ir} = (\pi_1')c_{ir}'$, where $c_{ir}' \in \mathcal{O}_{v'}$.
Hence we have proved that π_1' is a factor of each c_{ij}, but one can assume from the beginning that at least one c_{ij} is a unit for any $i = 1, 2, ..., f$. Thus, the only possibility is that $c_{ij} = 0$ for $i = 1, 2, \ldots, f$ and $j = 0, 1, \ldots, e-1$. This proves that $\mathcal{O}_v$ is in fact free $\mathcal{O}_{v'}$-module of rank ef.
Let $\alpha \in K$. Then does exit $s \in \mathcal{O}_{v'}$ such that $s\alpha \in \mathcal{O}_v$. So we get:

$$s\alpha = \sum_{j=0}^{e-1} \sum_{i=1}^{f} c_{ij} v_i \pi_1^j,\ c_{ij} \in \mathcal{O}_{v'},$$

or

$$\alpha = \sum_{j=0}^{e-1} \sum_{i=1}^{f} (\frac{c_{ij}}{s}) v_i \pi_1^j,\ \frac{c_{ij}}{s} \in K'.$$

Moreover, this expression is unique. Hence, $[K : K'] = ef$. All the other statements follows from [Se] (Proposition 3, p. 28). □

3 The general case

Proposition 3.1. *Let (K, v) be a discrete rank t valued field and $w_1, w_2, ..., w_t$ be its principal valuations on the fields K, $\overline{K}_{w_1}, \overline{K}_{w_2}, ..., \overline{K}_{w_{t-1}}$ respectively. Then, all the valued fields (K, w_1), $(\overline{K}_{w_1}, w_2), ..., (\overline{K}_{w_{t-1}}, w_t)$ are complete if and only if (K, v) is strongly complete.*

Proof: It is enough to see that a sequence $\{y_n\}$ of elements in K (or in $\overline{K}_{w_{i-1}}$, $i = 2, 3, ..., t$) is bounded relative to v (or to v_{i-1}) if and only if it is bounded relative to w_1 (or to w_i, $i = 2, 3, ..., t$). In particular, the topology of (K, v) (or of $(\overline{K}_{w_{i-1}}, v_{i-1})$) is the same like the topology of (K, w_1) (or of $(\overline{K}_{w_{i-1}}, w_i)$), with respect to their own valuations. $\square$

Theorem 3.1. *Let $(K', v') \subset (K, v)$ be an extension of discrete rank t valued fields such that $[G_v : G_{v'}] = e < \infty$, where G_v, $G_{v'}$ are the value groups of (K, v) and (K', v') respectively. Let $\overline{K}_v$ and $\overline{K'}_{v'}$ be the residue fields of (K, v) and (K', v') respectively. If $[\overline{K}_v : \overline{K'}_{v'}] = f < \infty$ and if (K', v') is a strongly complete valued field, then*

a) K/K' is a finite extension with $[K : K'] = ef$, and

b) K is also strongly complete.

Proof: We use mathematical induction on t. For $t = 1$, the statement of Theorem 3.1 becomes exactly the statement of Theorem 2.1. Suppose now that the statement of Theorem 3.1 was proved for any rank $= 1, 2, \ldots, t-1$. As above, we consider the marginal valuations w_1, w'_1 of v and v' respectively. We also consider the auxiliary valuations v_1 and v'_1 of (K, v) and (K', v') respectively. Since $G_v = \mathbb{Z} \times \cdots \times \mathbb{Z}$, t-times and since both valued fields have the same rank t, one has that $G_{v'} = e_1\mathbb{Z} \times \cdots \times e_t\mathbb{Z}$, where $e_1 e_2 \cdots e_t = e$. Let $\pi'_1, \pi'_2, \ldots, \pi'_t$ in $\mathcal{O}_{v'}$ such that $v'(\pi'_i) = (0, \ldots, 0, e_i, 0, \ldots, 0) \in \mathbb{Z}^t$. Since the residue valued field $(\overline{K'}_{w'_1}, v'_1)$ has the principal valuations $w'_2, \ldots, w'_t$, it is strongly complete. In addition, the rank of $(\overline{K}_{w_1}, v_1)$ is $t-1$ and $[G_{v_1} : G_{v'_1}] = e_2 e_3 \cdots e_t$. Since the residue fields of $(\overline{K'}_{w'_1}, v'_1)$ and $(\overline{K}_{w_1}, v_1)$ are $\overline{K'}_{v'}$ and $\overline{K}_v$ respectively, applying the mathematical induction hypothesis, one gets that $[\overline{K}_{w_1} : \overline{K'}_{w'_1}] = e_2 e_3 \cdots e_t f$ and that $\overline{K}_{w_1}$ is strongly complete. We now consider the extension $(K', w'_1) \subset (K, w_1)$ of discrete rank 1 valued fields. Since (K', w'_1) is complete, $[G_{w_1} : G_{w'_1}] = e_1$ and $[\overline{K}_{w_1} : \overline{K'}_{w'_1}] = e_2 e_3 \cdots e_t f$, Theorem 2.1 says that (K, w_1) is also complete and $[K : K'] = e_1 e_2 \cdots e_t f = ef$, etc. $\square$

Since the valuations v and v' are completly determined by the couples (w_1, v_1) and (w'_1, v'_1) respectively, using the above mathematical induction and Theorem 2.1 we get the unicity of v (relative to v'), i.e. we obtain the following result:

Corollary 3.1. *Let $(K', v') \subset (K, v)$ be a finite extension of discrete rank t valued fields and (K', v') be strongly complete. Then v is the unique extension of v' to K and $ef = n = [K : K']$.*

Remark 3.1. *If $k \subset K' \subset K = k((X))$ and K' is complete we obtain the main result of [NaP]. If $k \nsubseteq K' \subset K$, but K' is complete one obtains the main result of [Nas].*

Remark 3.2. *If instead of the condition $[\overline{K}_v : \overline{K'}_{v'}] < \infty$ we ask only that $\overline{K'}_{v'} \subset \overline{K}_v$ to be algebraic but not necessarily finite, the extension $K' \subset K$ is not in general algebraic. This can be proved by using the same ideas like those used by R. Gilmer in [Gi]. For instance, if $\overline{\mathbb{Q}}$ is an algebraic closure of $\mathbb{Q}$ then the extension $\mathbb{Q}((X)) \subset \overline{\mathbb{Q}}((X))$ contains transcendental elements. One of them, for instance is $\alpha = \sum_{n=1}^{\infty} 2^{(\frac{1}{2^{n!}})} X^{n!}$.*

Example 3.1. *Let $\mathbb{C}$ be the field of complex numbers and let $\mathbb{R}$ be its subfield of real numbers. Let $K = \mathbb{C}((X))((Y))$ be the field of Laurent series $s = \sum\limits_{(i,j)\succeq(i_0,j_0)} a_{ij} X^i Y^j$, lexicographically ordered relative to $(i, j) \in \mathbb{Z}\times\mathbb{Z}$, where i_0, $j_0 \in \mathbb{Z}$ and $a_{ij} \in \mathbb{C}$. Let v be the natural valuation of rank 2 on K (see Example 1.1) and let $K' = \mathbb{R}((X^2))((Y^3))$ be the analogous subfield of K. Then Theorem 3.1 says that $[K : K'] = 12$.*

Acknowledgements

We are thankful to Dr. N. Popescu (Institute of Mathematics of the Romanian Academy) for many useful discussions and to Dr. T. Dumitrescu (University of Bucharest) for making clear to us the situation mentioned in Remark 3.2. We are also grateful to the referee for some suggestions with respect to the general presentation of this paper.

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Received: 10.04.2008.

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