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On injectivity and p-injectivity, V.

by
Roger Yue Chi Ming

Dedicated to Chottoo Ibrahimsah

Abstract

This note contains the following results for a ring A: (1) A is von Neumann regular if, and only if, every divisible singular left A-module is p-injective and every p-injective left A-module is flat if, and only if, every simple right A-module and every divisible singular left A-module are flat; (2) If A is a left YJ-injective ring whose simple left modules are either YJ-injective or projective and g: $Q \to$ M is an epimorphism of left A-modules Q, M, where $_A$Q is CE-injective, then Z(M), the singular submodule of M, is a direct summand of M; (3) A is semi-simple, Artinian if, and only if, every semi-simple left A-module is quasi-injective and p-injective; (4) If every maximal left ideal of A is either injective or a two-sided ideal of A and every simple left A-module is YJ-injective, then A is either strongly regular or left self-injective regular with non-zero socle; (5) A right Noetherian, fully left idempotent ring is biregular; (6) A is strongly regular if, and only if, A is a reduced left p.p. ring having a classical left quotient ring such that for every proper principal left ideal P of A, r(P)$\neq$ 0; (7) A ring whose p-injective modules are injective and flat must be quasi-Frobenius; (8) If every essential left ideal of A is an idempotent two-sided ideal of A, then the centre of A is von Neumann regular.

Key Words: Flat, p-injective, YJ-injective, von Neumann regular, quasi-Frobenius.
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1 Introduction

Throughout, A denotes an associative ring with identity and A-modules are unital. J, Z, Y will stand respectively for the Jacobson radical, the left singular ideal and the right singular ideal of A. A is called semi-primitive (resp.(a) left non-singular; (b) right non-singular) if J=0 (resp. (a) Z=0; (b) Y=0). For any left A-module M, Z(M)=$\{y \in M \setminus l(z)$ is an essential left ideal of A$\}$ is the singular submodule of M. $_AM$ is called singular (resp.non-singular) if Z(M)=M (resp.Z(M)=0).

Thus Z= $Z(_AA)$ and Y= $Z(A_A)$. Following C.Faith [6], we write "A is VNR" if A is a von Neumann regular ring. It is well-known that A is VNR iff every left (right) A-module is flat (M.Harada (1956); M.Auslander (1957)). This remains true if "flat" is replaced by "p-injective" (cf. [1], [11], [16], [19]). Recall that (a) A right A-module M is p-injective if, for every principal right ideal P of A, any right A-homomorphism of P into M extends to one of A into M; (b) M_A is YJ-injective if, for any $0 \neq a \in A$ there exists a positive integer n such that $a^n \neq 0$ and every right A-homomorphism of a^nA into M extends to one of A into M ([16], [25], [26], [30], [32]). If A_A is p-injective (resp.YJ-injective), then A is called a right p-injective (resp.YJ-injective) ring. Left p-injectivity and YJ-injectivity are similarly defined. If A is right YJ-injective, then Y=J (this is the origin of our notation) ([25], [30]). Also, A is right YJ-injective iff for every $0 \neq a \in A$, there exists a positive integer n such that Aa^n s a non-zero left annihilator [25, Lemma 3] (cf. also [32, Corollary 2]).

Note that Harada-Auslander's characterization of VNR rings may be weakened as follows: A is VNR iff every cyclic singular left A-module is flat [21, Theorem 5]. Of course, flatness and p-injectivity are distinct concepts. During the last thirty years, motivated by the study of flat modules over non-VNR rings, various authors have studied p-injectivity and YJ-injectivity (also called GP-injectivity) over rings not necessarily VNR.

The next result is a sequel to [30, Theorem 16]. As usual, A is called a left (resp.right) SF-ring if every simple left (resp.right) A-module is flat.

Theorem 1. *The following conditions are equivalent:*

1. *A is VNR;*
2. *Every divisible singular left A-module is p-injective and every p-injective left A-module is flat;*
3. *Every divisible singular left A-module is p-injective and every p-injective right A-module is flat;*
4. *A is a right SF-ring whose divisible singular left modules are flat;*
5. *A is a left SF-ring whose divisible singular left modules are flat.*

Proof: (1) implies through (5) evidently.

Assume (2). Then every finitely generated left ideal of A is a left annihilator [9, Corollary 2.5] which implies that A is right p-injective. Then every non-zero-divisor is invertible in A and consequently, every left (or right) A-module is divisible [29, Proposition 1]. By hypothesis, every singular left A-module is p-injective which implies that every singular left A-module is flat. By [21, Theorem 5], A is VNR and (2) implies (1).

Assume (3). Then A is a left p-injective ring. For any injective left A-module M, every submodule N of M, it is sufficient to show that M/N is a p-injective left A-module in order that A be a left p.p.ring [20, Remark 2(iii)]. If E(N) denotes the injective hull of $_A$N in M, then M=E(N)$\oplus$ Q for some injective submodule Q of M. Now $_A$E(N)/N is singular, divisible and by hypothesis, E(N)/N is p-injective. Also, (Q+N)/N $\approx$ Q/(Q $\cap$ N)=Q and therefore, M/N=E(N)/N $\oplus$ (Q+N)/N is a p-injective left A-module. Consequently, every quotient of a p-injective left A-module is p-injective [20, Remark 2(ii)]. For any cyclic left A-module C=Ac, C $\approx$ A/l(c) is p-injective and hence (3) implies (1) by [19, Lemma 2].

Assume (4). By [30, Proposition 4], every non-zero-divisor is invertible in A and every left (or right) A-module is divisible. Consequently, every singular left A-module is flat and (4) implies (1) by [21, Theorem 5].

Similarly, (5) implies (1). □

Recall the following generalization of quasi-injective modules introduced in [24]: A left A-module M is called CE-injective if, for any left submodule C of M containing a non-zero complement left submodule of M, every left A-homomorphism of C into M extends to an endomorphism of $_A$M.

Proposition 2. *Let A be a left YJ-injective ring whose simple left modules are either YJ-injective or projective. If g: Q $\to$ M is an epimorphism of left A-modules Q, M with $_A$Q CE-injective, then Z(M), the singular submodule of M, is a direct summand of M.*

Proof: Since A is left YJ-injective, then J=Z[25, p.103]. Since every simple left A-module is either YJ-injective or projective, then J $\cap$ Z=0 [26, Proposition 8]. Consequently, Z=J=0. Since g^{-1} (Z(M))= Cl (ker g) is a complement left submodule of Q by [18, Theorem 4(3)], then g^{-1}(Z(M)) is a direct summand of Q by [24, Proposition 1]. Therefore Q= g^{-1} (Z(M))$\oplus$ N for some submodule N of Q. It follows that M=g(Q)=Z(M)$\oplus$g(N), where g(N)$\approx$N. □

Remark 1. Proposition 2 holds if "left YJ-injective" is replaced by "MELT". Recall that A is MELT (resp.MERT) if, for any maximal essential left (resp.right) ideal M of A (if it exists), M is an ideal of A. (An ideal of A will always mean a two-sided ideal of A.)

Also, A is ELT (resp.ERT) if every essential left (resp.right) ideal of A is an ideal of A.

Following Michler-Villamayor [12], a left A-module M is called semi-simple if the intersection of all maximal submodules of M is zero. Thus A is semi-simple iff A is semi-primitive. It is well-known that A is a left V-ring iff every left A-module is semi-simple [12, Theorem 2.1].

The next theorem is a sequel to [26, Theorem 7] and Condition (2) slightly weakens Condition (2) in [12, Theorem 3.2].

Theorem 3. *The following conditions are equivalent:*

1. *A is semi-simple Artinian;*
2. *Every semi-simple left A-module is quasi-injective and p-injective;*
3. *Every semi-simple left A-module is quasi-injective and every simple left A-module is p-injective;*
4. *A is a semi-prime MERT right Goldie ring whose simple right modules are YJ-injective or flat;*
5. *A is a semi-prime MERT ring with maximum condition on left and right annihilators such that every simple right A-module is either YJ-injective or flat.*

Proof: It is evident that (1) implies (2) and (4) while (4) implies (5). It is also clear that (2) implies (3).

Assume (3). We first prove that every left ideal of A is semi-simple. Suppose the contrary: Let L be a left ideal of A which is not semi-simple. Then there exists $0 \neq y \in L$ such that y belongs to every maximal left subideal of L. By Zorn's Lemma, the set of all left subideals K of Ay such that $y \notin K$ has a maximal member V. Then $y \notin V$ and Ay/V is a simple left A-module and therefore p-injective. Let $g: Ay \to Ay/V$ be the canonical homomorphism. Then g extends to f: $A \to Ay/V$. Restrict f to $h: L \to Ay/V$. Then L/ker $h \approx Ay/V$, which proves that ker h is a maximal subideal of L. Since ker $h \cap Ay =$ ker g, $y \in Ay$, $y \notin$ ker g, then $y \notin$ ker h. This contradiction proves that every left ideal of A is semi-simple and by hypothesis, every left ideal of A is quasi-injective. Since J=0 by [20, Lemma 1], then A is a left and right self-injective, left and right V-ring of bounded index. By [12, Theorem 2.1], every left A-module is semi-simple and hence quasi-injective. Thus (3) implies (1).

Assume (5). Suppose there exists a maximal right ideal M of A which is not a direct summand of A_A. Then M is an ideal of A (because A is MERT). By [10, Theorem], M contains a non-zero-divisor c. If A/M_A is YJ-injective, there exists a positive integer n such that any right A-homomorphism of c^nA into A/M extends to one of A into A/M. Define the right A-homomorphism $g: c^nA \to A/M$ by $g(c^na)=a+M$ for all $a \in A$. Then $g(c^n)=yc^n+M$ for some $y \in A$. But $g(c^n)=1+M$ which implies that $1-yc^n \in M$, whence $1 \in M$ (because M is an ideal of A), contradicting $M \neq A$. If A/M_A is flat, since $c \in M$, $c=dc$ for some $d \in M$ [3, p.458]. Then (1-d) c=0 implies that $1=d \in M$, again a contradiction! We have proved that every maximal right ideal of A is a direct summand of A_A and hence (5) implies (1). □

Note that in condition (5) above, the term "semi-prime" is not superfluous, as shown by the following example.

Example Let A denote the 2x2 upper triangular matrix ring over a field. Then A is a P.I. ERT (and ELT) right (and left) Artinian, hereditary ring whose simple one-sided modules are either injective or projective. But A is not semi-prime ($J \neq 0$ with $J^2 = 0$). This example shows that if A is a ring whose simple left modules are either injective or projective and every maximal left ideal of A is either an ideal of A or an injective left A-module, then A needs not be VNR.

Proposition 4. *Let A be a ring such that each maximal left ideal is either injective or an ideal of A. If every simple left A-module is YJ-injective, then A is either strongly regular or left self-injective regular with non-zero socle.*

Proof: First suppose that every maximal left ideal of A is an ideal of A. Then A is strongly regular by [28, Theorem 1]. Now suppose there exists one maximal left ideal M of A which is not an ideal of A. By hypothesis, ${}_AM$ is injective. Then A is left self-injective by [27, Lemma 4]. Now J=0 by [28, Lemma 1] which implies that A is VNR with non-zero socle. □

Remark 2. Proposition 4 remains valid if we replace "every simple left A-module" by "every simple right A-module".

Recall that A is biregular if, for every a∈A, AaA is generated by a central idempotent. Biregular rings generalize effectively strongly regular rings and simple rings.

Proposition 5. *Let A be a right Noetherian, fully left idempotent ring. Then every ideal of A is generated by a central idempotent. In particular, A is a biregular ring.*

Proof: Let T denote an ideal of A. If t∈T, $At=(At)^2$ implies that t=dt for some $d \in AtA \subseteq T$. Therefore A/T_A is flat [3, p.458]. Since T_A is finitely generated, A/T is a finitely related flat right A-module which implies that A/T_A is projective. It follows that T=eA, $e=e^2 \in A$. Since A is semi-prime and T is an ideal of A, e is central in A. In particular, A is biregular. □

Applying [20, Lemma 1], we get

Corollary 5.1. *A right Noetherian ring whose simple left modules are p-injective must be biregular.*

Corollary 5.2. *If A is a right Noetherian ring such that for every essential right ideal R of A which is an ideal of A, A/R_A is flat, then A is biregular.*

Corollary 5.3. *A prime right Noetherian fully left idempotent ring is simple. (Such rings need not be Artinian).*

The proof of Proposition 5 yields

Remark 3. A is biregular iff A is a fully left idempotent ring such that for each a∈A, AaA is a finitely generated right ideal of A.

Proposition 6. *The following conditions are equivalent:*

1. *A is strongly regular;*
2. *A is a reduced left p.p.ring having a classical left quotient ring such that for every proper principal left ideal P of A, r(P)$\neq$ 0;*
3. *A is a reduced ring whose injective right modules are flat.*

Proof: (1) implies (2) and (3) obviously.

Assume (2). Since A is reduced, Z=0. Let c denote a non-zero-divisor of A. Since A has a classical left quotient ring, then Ac is an essential left ideal of A. If Ac$\neq$A, let 0 $\neq$z$\in$r(Ac). Then Ac $\subseteq$l(z) implies that z$\in$Z=0, a contradiction! Therefore Ac= A which implies that c is left invertible and hence invertible in A. Since A is reduced, left p.p., for every a$\in$A, there exists a central idempotent e and a non-zero-divisor u such that a=eu. Therefore $e=a^{-1}u$ and $e=e^2$ yields $au^{-1}=au^{-1}au^{-1}$, whence $a=au^{-1}a$, showing that A is VNR. Therefore (2) implies (1).

Assume (3). Then A is left YJ-injective by [9, Corollary 2.5], [25, Lemma 3]. Now A is strongly regular by [25, Lemma 5]. Thus (3) implies (1). □

Corollary 6.1. *A commutative p.p. ring whose proper principal ideals have non-zero annihilators must be VNR.*

Remark 4. The proof of Proposition 6 shows that strongly regular rings must be unit-regular (cf.also the proof of [19, Proposition 1]).

Remark 5. Let A be a left p.p. ring which is left YJ-injective. For any 0 $\neq$a$\in$A, by [25, Lemma 3], there exists a positive integer n such that a^nA is a non-zero right annihilator. Since A is left p.p., $l(a^nA)=l(a^n)$ is a direct summand of ${}_AA$. Consequently, $a^nA= r(l(a^nA))=r(l(a^n))$ is a direct summand of A_A. Therefore every right A-module is YJ-injective and by [32, Theorem 9], A is VNR.

Proposition 7. *Let A be a ring whose p-injective left modules are injective and flat. Then A is quasi-Frobenius.*

Proof: Since a direct sum of left A-modules is p-injective iff every direct summand is p-injective, then any direct sum of injective left A-modules is p-injective and, by hypothesis, is therefore injective. By [5, Theorem 20.1], A is left Noetherian. Since every injective left A-module is flat, then A is quasi-Frobenius by [17, Lemma 1.3]. Recall that a left A-module M is f-injective if, for any finitely generated left ideal F of A, every left A-homomorphism of F into M extends to one of A into M. □

The proof of Proposition 7 yields

Theorem 8. *The following conditions are equivalent:*

1. *A is quasi-Frobenius;*
2. *Every f-injective left A-module is injective and flat.*

The following result also holds.

Theorem 9. *The following conditions are equivalent:*

1. *A is a left and right principal ideal ring which is quasi-Frobenius;*
2. *A has the following properties: (a) every p-injective left A-module is injective and flat; (b) every finitely generated one-sided ideal of A is the annihilator of an element of A. (Condition 2(b) implies that every finitely generated one-sided ideal of A is principal.)*

Question 1. If A is quasi-Frobenius, (a) Are p-injective left A-modules injective? (b) Are p-injective left A-modules flat?

Remark 6. Theorem 9 shows that "every finitely generated left ideal of A is a left annihilator" is not equivalent to "every finitely generated left ideal of A is the left annihilator of on element of A".

An ELT ring whose simple left modules are either injective or projective needs not be VNR as shown by the given example. We know that if every simple left A-module is either p-injective or projective, then every essential left ideal of A is idempotent. Consequently, the ring A considered in the next result needs not be VNR (not even semi-prime).

Proposition 10. *Let A be a ring whose essential left ideals are idempotent two-sided ideals of A. Then the centre of A is VNR.*

Proof: Let C denote the centre of A. For any $c \in C$, let K be a complement left ideal of A such that $L = Ac \oplus K$ is an essential left ideal of A. Then $c \in L^2 = L$ implies that $c = \sum_{i=1}^{n} (a_i c + k_i)(b_i c + t_i)$, a_i, $b_i \in A$, k_i, $t_i \in K$, whence $c - \sum_{i=1}^{n} (a_i c + k_i) b_i c = \sum_{i=1}^{n} (a_i c + k_i) t_i \in Ac \cap K = 0$. Now $k_i b_i \in L$ (in as much as L is an ideal of A) implies that $k_i b_i = u_i c + s_i$, $u_i \in A$, $s_i \in K$ and therefore $c - \sum_{i=1}^{n} a_i c b_i c = \sum_{i=1}^{n} k_i b_i c = \sum_{i=1}^{n} (u_i c + s_i) c$ which yields $c - \sum_{i=1}^{n} a_i c b_i c - \sum_{i=1}^{n} u_i c^2 = \sum_{i=1}^{n} s_i c = \sum_{i=1}^{n} c s_i \in Ac \cap K = 0$. Therefore $c = c(\sum_{i=1}^{n} (a_i b_i + u_i))c$, where $d = \sum_{i=1}^{n} (a_i bi + u_i) \in A$. Then $c(c^2 d^3)c = (cdc)dcdc = cdcdc = c$. Again, $c^2 d = dc^2 = cdc = c$ and hence, for every $b \in A$, $dc^2 b = cb = bc = bdc^2 = c^2 bd$. Therefore d^3 commutes with $c^2 b$ and we have $c^2 d^3 b = d^3 c^2 b = c^2 b d^3 = b c^2 d^3$, showing that $c^2 d^3 \in C$. With $t = c^2 d^3$, we get $ctc = c$. Therefore C is VNR. □

Corollary 10.1. *A commutative ring is VNR iff every essential ideal is idempotent.*

Question 2. If every essential left ideal of A is idempotent, is the centre of A VNR?

The next result may be added to [1, Theorem 2.2] (cf.also[14, p.577]).

Proposition 11. *The following conditions are equivalent:*

1. *Every simple left A-module is either injective or projective;*
2. *Any left A-module which contains no simple projective left submodule is semi-simple.*

Proof: Assume (1). Suppose there exists a left A-module M which contains no simple projective submodule such that W, the intersection of all maximal submodules of M, is non-zero. Let $0 \neq y \in W$. By Zorn's Lemma, the set of all left submodules of M not containing y has a maximal member Q. Then $y \notin Q$ and y belongs to every submodule of M which strictly contains Q. Let T denote the intersection of all left submodules N of M such that $Q \subset N$ (strict inclusion). Then $y \in T$ and ${}_AT/Q$ is simple which, by hypothesis, cannot be projective. Then $M/Q = T/Q \oplus U/Q$, which implies that $y \notin U$. Therefore U=Q and Q is a maximal submodule of M. This contradicts the hypothesis that y belongs to every maximal submodule of M. We have shown that (1) implies (2).

Assume (2). Let V be a simple non-projective left A-module, I a proper essential left ideal of A, g: $I \to V$ a non-zero left A-homomorphism. Then ${}_AI/K \approx {}_AV$, where K=ker g is a maximal left submodule of I. Suppose that $K \neq K^*$, where K^* denotes the intersection of all maximal left ideals of A containing K. Then ${}_AA/K$ is not semi-simple and by hypothesis, contains a simple projective submodule P/K. Now $P = K \oplus U$, where U is a minimal projective left ideal. Since $U \subseteq I$ (I being essential), $I = K \oplus U$ which implies that $V \approx U$ is projective, a contradiction! Therefore $K = K^*$ and there exists a maximal left ideal M of A such that $K \subseteq M$ but $I \nsubseteq M$. Since ${}_AI/K$ is simple, then $M \cap I = K$ which yields $A/M = (M+I)/M \approx I/(M \cap I) = I/K \approx V$, showing that g may be extended to a left A-homomorphism of A into V. This proves that ${}_AV$ is injective and (2) implies (1). □

Note that (a) MELT fully idempotent rings need not be VNR [31, Theorem 1]; (b) ELT fully idempotent rings are VNR [27, Proposition 8]. (Consequently, MELT generalizes effectively ELT.) However, we have

Proposition 12. *Let A be a MELT, fully idempotent ring such that for any maximal essential left ideal M of A, A/M_A is flat. Then A is an ELT, VNR ring.*

Proof: Let M be a maximal left ideal of A which is essential in ${}_AA$. By hypothesis, M is an ideal of A and A/M_A is flat. Then ${}_AA/M$ must be injective (cf. the proof of [30, Theorem 8]). Therefore, every simple left A-module is either injective or projective which implies that every proper essential left ideal of A is an intersection of maximal left ideals of A. Since A is MELT, then A is ELT. By [27, Proposition 8], A is VNR. □

Recall that

1. MELT left and right V-rings are ELT, unit-regular;
2. A is a right V-ring iff for any right ideal R of A and each maximal subideal Y of R, there is a maximal right ideal M of A with M∩R=Y(cf. [13, Theorem 22.1]);
3. A is VNR iff for any proper principal right ideal P of A, there exist an idempotent e and a p-injective maximal right ideal M of A such that P=eA∩M;
4. Every simple right A-module is p-injective iff for any principal right ideal I of A, any maximal right subideal K of I, there exists a maximal right ideal M of A such that K=I∩M [14, p.578];
5. A is unit-regular iff A is a directly finite ring such that for any 0 ≠a∈A, either a is right invertible or there exist a non-trivial idempotent e and a left regular element d such that a=ed.

Finally, we note that

1. Chen, Zhou, Zhu recently showed that YJ-injectivity effectively generalizes p-injectivity (even for rings) [4];
2. For a left uniserial ring A, the following are equivalent: (a) A is right p-injective; (b) A is right YJ-injective; (c) J=Y (cf.W.K.Nicholson, M.F Yousif: Bull. Austral. Math. Soc. (87(1994), 513-518).
3. More than one hundred papers have been published by various authors on p-injectivity and YJ-injectivity (in addition to more than fifty of my own papers).

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Universite Paris VII-Denis Diderot
UFR De Mathematiques-UMR 9994 CNRS
2, Place Jussieu,
75 251 Paris CEDEX 05,
France