

Estimation of the Mean Time to Failure relative to a Class of Loss Functions

by

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Abstract

In this paper we consider the estimation of the mean time to failure using a weighted loss function that reflects fit and precision of estimation. The weighted loss function we propose here is an extension of weighted balanced loss function used in Rodrigues and Zellner [7] and a special case of the one used in Chung et al [1]. As in the previously mentioned article, we will prove that optimal point estimates relative to our weighted loss function are linear combination between Bayesian and non-Bayesian estimates. Examples based on non-informative and informative priors for evaluating posterior expected loss associated to alternative estimates are given. The results obtained generalize the results from Ciumara et al. [3].

Key Words: Weighted balanced loss function, optimal estimates, Bayesian estimates, informative and non-informative priors.

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1 Introduction

A goodness of fit criterion such as the sum of squared residuals in regression problems leads to an estimate which gives good fit and unbiased estimator. Rodrigues and Zellner [7] argued that it may not be as precise as an estimator which is biased.

Thus there is a need to provide a framework which combines the goodness of fit and precision of estimation formally. Also, Rodrigues and Zellner [7] indicated that other measures of goodness of fit and precision of estimation could be incorporated in more general functional forms.

Rodrigues and Zellner [7] introduced and analyzed the balanced function

$$\tilde{L}_B(\hat{\theta}, \theta) = w \frac{\sum_{i=1}^n (X_i - \hat{\theta})^2}{n} + (1 - w) (\hat{\theta} - \theta)^2$$

and the two-parts weighted balanced loss function

$$\tilde{L}_{WB}(\hat{\theta}, \theta) = w \frac{\sum_{i=1}^n (X_i - \hat{\theta})^2}{n\hat{\theta}^2} + (1-w) \frac{(\hat{\theta} - \theta)^2}{\hat{\theta}^2}$$

where $\hat{\theta}$ is some estimate of θ and the given weight $w \in [0, 1]$. Generalizations of this loss function could be found in Ciumara et al. [3] and Chung et al. [1].

The first term in the right-hand-side of these functions represents a quadratic measure of goodness-of-fit and the second term is a squared error measure of precision.

In weighted balanced loss function expression, relative squared errors are used instead of squared error terms in both parts of this loss function.

If $w = 0$ we get squared error or squared relative error loss function, and if $w = 1$, we get a pure goodness-of-fit criterion.

A thorough treatment of Bayesian, non-Bayesian and optimal estimates of parameters, could be found in Preda [5,6], Zellner [8] and Martz and Waller [4].

In this paper we focus on the three-parts balanced and weighted balanced loss function considered in Ciumara et al. [3]. In Section 2 we use the proposed three-parts loss functions in order to give optimal estimate of the mean time to failure. We evaluate posterior expected loss for various estimates and compare them to these of optimal estimates.

We apply the results obtained in Section 2 for a non-informative prior, in Section 3, and for an informative prior, in Section 4.

Finally, we give a simple numerical example and some concluding remarks.

2 Three-parts weighted loss functions and optimal estimates of the mean time to failure

Let a three-parts balanced and weighted balanced loss functions be L_B

$$L_B(\hat{\theta}, \theta) = w_1 s^2 + w_2 (\hat{\theta} - \bar{X})^2 + w_3 (\hat{\theta} - \theta)^2$$

and, respectively, L_{WB} ,

$$L_{WB}(\hat{\theta}, \theta) = w_1 \frac{s^2}{\hat{\theta}^2} + w_2 \frac{(\hat{\theta} - \bar{X})^2}{\hat{\theta}^2} + w_3 \frac{(\hat{\theta} - \theta)^2}{\hat{\theta}^2},$$

where the weights satisfy $w_1, w_2, w_3 \in [0, 1]$ and $w_1 + w_2 + w_3 = 1$ (Ciumara et al. [3]).

We should note however that the balanced loss function and the weighted balanced loss function introduced by Rodrigues and Zellner [7] could be expressed as

$$\tilde{L}_B(\hat{\theta}, \theta) = w \left(s^2 + (\hat{\theta} - \bar{X})^2 \right) + (1 - w) (\hat{\theta} - \theta)^2$$

and, respectively,

$$\tilde{L}_{WB}(\hat{\theta}, \theta) = w \frac{s^2 + (\hat{\theta} - \bar{X})^2}{\hat{\theta}^2} + (1 - w) \frac{(\hat{\theta} - \theta)^2}{\hat{\theta}^2}.$$

Therefore, the three-parts loss functions proposed here appear naturally by imposing different weights for s^2 and $(\hat{\theta} - \bar{X})^2$, components of the measure of goodness-of-fit.

Definition 2.1. (Rodrigues and Zellner [7]) *Conditional on the data and prior information, an estimate θ_{**} of θ is optimal if it minimizes the posterior expectation of a given loss function.*

We denote by

$$L_1(\hat{\theta}) = w_1 \frac{s^2}{\hat{\theta}^2} + w_2 \frac{(\hat{\theta} - \bar{X})^2}{\hat{\theta}^2}$$

and

$$L_2(\hat{\theta}, \theta) = \frac{(\hat{\theta} - \theta)^2}{\hat{\theta}^2}.$$

The following result enables to prove the main theorem of this section.

Lemma 2.1. *The posterior expectations of L_2 is*

$$E(L_2) = \frac{v}{E(\theta^2)} + \frac{\bar{\theta}^2}{E(\theta^2)} \frac{(\hat{\theta} - \hat{\theta}_2)^2}{\hat{\theta}^2},$$

where $\bar{\theta}$ is the posterior mean of θ , $v = E((\theta - \bar{\theta})^2)$ the posterior variance and $\hat{\theta}_2 = \frac{E(\theta^2)}{\bar{\theta}}$.

Proof: Considering the expression of L_2 , we obtain

$$E(L_2) = E\left(\frac{(\hat{\theta} - \theta)^2}{\hat{\theta}^2}\right) = \frac{(\hat{\theta} - \bar{\theta})^2 + 2(\hat{\theta} - \bar{\theta})E(\bar{\theta} - \theta) + E((\bar{\theta} - \theta)^2)}{\hat{\theta}^2}.$$

Since $E(\theta) = \bar{\theta}$, we get

$$E(L_2) = \frac{(\hat{\theta} - \bar{\theta})^2 + v}{\hat{\theta}^2}.$$

But $v = E((\theta - \bar{\theta})^2) = E(\theta^2) - \bar{\theta}^2$, thus $E(\theta^2) = v + \bar{\theta}^2$. Then, since $\frac{v}{\bar{\theta}} + \bar{\theta} = \frac{v + \bar{\theta}^2}{\bar{\theta}} = \frac{E(\theta^2)}{\bar{\theta}} = \hat{\theta}_2$, we get

$$E(L_2) = \frac{v}{E(\theta^2)} + \frac{\bar{\theta}^2}{E(\theta^2)} \frac{(\hat{\theta} - \hat{\theta}_2)^2}{\hat{\theta}^2}.$$

□

Now we could present the main result.

Theorem 2.1. *For any posterior density $\pi(\theta|X_1, \dots, X_n)$, we have:*

i) the optimal estimate for θ is given by

$$\theta_{**} = \frac{w_1 s^2 + w_2 \bar{X}^2 + w_3 \bar{\theta} \hat{\theta}_2}{w_2 \bar{X} + w_3 \bar{\theta}} = \frac{w_2 \bar{X} \hat{\theta}_1 + w_3 \bar{\theta} \hat{\theta}_2}{w_2 \bar{X} + w_3 \bar{\theta}},$$

where $\hat{\theta}_1$ is the value of $\hat{\theta}$ that minimizes L_1 and $\hat{\theta}_2$ is the value of $\hat{\theta}$ that minimizes $E(L_2)$.

ii) the minimal posterior expected loss is given by

$$\begin{aligned} E(L_{WB}(\theta_{**}, \theta)) &= w_2 + w_3 - \frac{w_2 \bar{X} + w_3 \bar{\theta}}{\theta_{**}} = \\ &= 1 - \left(w_1 + \frac{w_2 \bar{X} + w_3 \bar{\theta}}{\theta_{**}} \right). \end{aligned}$$

Proof: i) Since $L_1(\hat{\theta}) = w_1 \frac{s^2}{\hat{\theta}^2} + w_2 \frac{(\hat{\theta} - \bar{X})^2}{\hat{\theta}^2}$, we get

$$\frac{\partial L_1(\hat{\theta})}{\partial \hat{\theta}} = \frac{2}{\hat{\theta}^3} \left[-w_1 s^2 + w_2 (\hat{\theta} - \bar{X}) \bar{X} \right].$$

Therefore, $\frac{\partial L_1(\hat{\theta})}{\partial \hat{\theta}} = 0$ implies $w_1 s^2 = w_2 (\hat{\theta} - \bar{X}) \bar{X}$, which leads to the value of $\hat{\theta}$ that minimizes L_1 ,

$$\hat{\theta}_1 = \frac{w_1 s^2 + w_2 \bar{X}^2}{w_2 \bar{X}}.$$

Obviously, from the previous relation we get $w_1 s^2 + w_2 \bar{X}^2 = w_2 \bar{X} \hat{\theta}_1$.

Note that $\hat{\theta}_1$ could be written as

$$\hat{\theta}_1 = \frac{\bar{X}}{1 - \lambda_{w_1, w_2}},$$

where $\lambda_{w_1, w_2} = \frac{w_1 s^2}{w_1 s^2 + w_2 \bar{X}^2}$.

Now, $L_2(\hat{\theta}, \theta) = \frac{(\hat{\theta} - \theta)^2}{\hat{\theta}^2}$ and from Lemma 2.1, we know that $E(L_2(\hat{\theta}, \theta)) = \frac{v}{E(\theta^2)} + \frac{\bar{\theta}}{E(\theta^2)} \frac{(\hat{\theta} - \hat{\theta}_2)^2}{\hat{\theta}^2}$. It results that $\frac{\partial E(L_2(\hat{\theta}, \theta))}{\partial \hat{\theta}} = 0$ for $\hat{\theta} = \frac{E(\theta^2)}{\bar{\theta}}$, that is, the value of $\hat{\theta}$ that minimizes the posterior expected loss $E(L_2(\hat{\theta}, \theta))$ is

$$\hat{\theta}_2 = \frac{E(\theta^2)}{\bar{\theta}}.$$

Moreover, since $L_{WB}(\hat{\theta}, \theta) = w_1 \frac{s^2}{\hat{\theta}^2} + w_2 \frac{(\hat{\theta} - \bar{X})^2}{\hat{\theta}^2} + w_3 \frac{(\hat{\theta} - \theta)^2}{\hat{\theta}^2}$, we get

$$E(L_{WB}(\hat{\theta}, \theta)) = w_1 \frac{s^2}{\hat{\theta}^2} + w_2 \frac{(\hat{\theta} - \bar{X})^2}{\hat{\theta}^2} + w_3 \left[1 - 2 \frac{\bar{\theta}}{\hat{\theta}} + \frac{E(\theta^2)}{\hat{\theta}^2} \right]$$

and, consequently,

$$\frac{\partial E(L_{WB}(\hat{\theta}, \theta))}{\partial \hat{\theta}} = \frac{2}{\hat{\theta}^3} \left[-w_1 s^2 + w_2 (\hat{\theta} - \bar{X}) \bar{X} + w_3 (\bar{\theta} \hat{\theta} - E(\theta^2)) \right].$$

Considering the above expression, $\frac{\partial E(L_{WB}(\hat{\theta}, \theta))}{\partial \hat{\theta}} = 0$ implies that $\hat{\theta} [w_2 \bar{X} + w_3 \bar{\theta}] = w_1 s^2 + w_2 \bar{X}^2 + w_3 E(\theta^2)$ which leads to the value of $\hat{\theta}$ that minimizes $E(L_{WB}(\hat{\theta}, \theta))$, that is, the optimal estimate of θ which is given by θ_{**} from i).

ii) The minimal posterior expected loss is given by $E(L_{WB}(\theta_{**}, \theta))$. After some calculations we get

$$E(L_{WB}(\hat{\theta}, \theta)) = 1 - w_1 - \frac{2}{\hat{\theta}}(w_2\bar{X} + w_3\bar{\theta}) + \frac{\theta_{**}(w_2\bar{X} + w_3\bar{\theta})}{\hat{\theta}^2}.$$

Therefore,

$$E(L_{WB}(\theta_{**}, \theta)) = w_2 + w_3 - \frac{w_2\bar{X} + w_3\bar{\theta}}{\theta_{**}},$$

exactly what we had to prove. $\square$

Remark 2.1. *We have*

$$L_1(\hat{\theta}) = w_2 \left(\lambda_{w_1, w_2} + (1 - \lambda_{w_1, w_2}) \frac{(\hat{\theta} - \hat{\theta}_1)^2}{\hat{\theta}^2} \right),$$

where $\lambda_{w_1, w_2} = \frac{\mu}{w_2}$, with $\mu = \frac{1}{n} \left(A - \frac{B^2}{C} \right)$, $A = nw_2$, $B = nw_2\bar{X}$ and $C = n(w_1s^2 + w_2\bar{X}^2)$.

Therefore, writing L_1 as above suggests to compare $\hat{\theta}$ to θ in a relative squared error sense and, consequently, motivates the loss function $L_{WB}(\hat{\theta}, \theta)$.

3 The exponential model with noninformative prior for θ

Here we consider the random variable X representing the time to failure, exponentially distributed. The probability density function is, in this case

$$f(x|\theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}},$$

with $0 \leq x < \infty$ and $0 < \theta < \infty$. Obviously, $E(X) = \theta$ is the mean time to failure and $Var(X) = \theta^2$.

As in Rodrigues and Zellner [7], the failure rate $\gamma = \frac{1}{\theta}$ is constant, therefore we analyze the exponential model. When the failure rate varies with time, we could employ other models, for example, Weibull model.

The usual noninformative prior for θ is proportional to $\frac{1}{\theta}$,

$$\pi(\theta) \sim \frac{1}{\theta}$$

(Preda [6]). Thus, for the exponential model, the posterior density is

$$\pi(\theta|x_1, \dots, x_n) = \frac{(n\bar{x})^n}{\Gamma(n)\theta^{n+1}} e^{-\frac{n\bar{x}}{\theta}}.$$

The posterior mean and posterior variance are given by

$$\bar{\theta} = E(\theta|x_1, \dots, x_n) = \frac{n\bar{x}}{n-1}$$

and, respectively,

$$\begin{aligned} v &= E(\theta^2|x_1, \dots, x_n) - (E(\theta|x_1, \dots, x_n))^2 = \\ &= \frac{(n\bar{x})^2}{(n-1)^2(n-2)} \end{aligned}$$

(Ciumara [2]).

Moreover,

$$\hat{\theta}_2 = \frac{E(\theta^2|x_1, \dots, x_n)}{\bar{\theta}} = \frac{n\bar{x}}{n-2}.$$

Remark 3.1. From Theorem 2.1. we have that

$$\begin{aligned} E(L_{WB}(\theta_{**}, \theta)) &= 1 - w_1 - \frac{w_2\bar{X} + w_3\bar{\theta}}{\theta_{**}} = \\ &= 1 - w_1 - \alpha, \end{aligned}$$

where $\alpha = \frac{w_2\bar{X} + w_3\bar{\theta}}{\theta_{**}}$.

For the noninformative prior mentioned before, we get

$$\theta_{**} = \frac{w_2}{w_2 + w_3 \frac{n}{n-1}} \hat{\theta}_1 + \frac{w_3 \frac{n}{n-1}}{w_2 + w_3 \frac{n}{n-1}} \hat{\theta}_2.$$

If we denote

$$\Sigma = \frac{w_2}{w_2 + w_3 \frac{n}{n-1}},$$

then

$$\theta_{**} = \Sigma \cdot \hat{\theta}_1 + (1 - \Sigma) \cdot \hat{\theta}_2.$$

Since $\hat{\theta}_1 = \frac{w_1 s^2 + w_2 \bar{X}^2}{w_2 \bar{X}}$ is a non-Bayesian estimate of θ and $\hat{\theta}_2 = \frac{E(\theta^2)}{\bar{\theta}}$ is a Bayesian estimate of the same parameter, it results that the optimal estimate of θ could be expressed as a linear combination of Bayesian and non-Bayesian estimates of θ .

Moreover, in this case, we derive α from $E(L_{WB}(\theta_{**}, \theta))$ and obtain

$$\alpha = \frac{\left(w_2 + w_3 \frac{n}{n-1}\right)^2}{w_1 \frac{s^2}{\bar{X}^2} + w_2 + w_3 \frac{n^2}{(n-1)(n-2)}}.$$

Now, it is of interest to compare $E(L_{WB}(\theta_{**}, \theta))$ with the posterior expected loss associated to $\hat{\theta}_1$ and/or $\hat{\theta}_2$. After some calculations, we get

$$\Delta = E(L_{WB}(\hat{\theta}_1, \theta)) - E(L_{WB}(\theta_{**}, \theta)) = \alpha \frac{(\hat{\theta}_1 - \theta_{**})^2}{\hat{\theta}_1^2}.$$

Because $\Sigma = \frac{w_2}{w_2 + w_3 \frac{n}{n-1}}$, we can express α as

$$\alpha = \frac{\left(\frac{w_2}{\Sigma}\right)^2}{u},$$

where

$$u = \frac{w_2}{1 - \lambda_{w_1, w_2}} + w_3 \frac{n^2}{(n-1)(n-2)}.$$

Therefore,

$$\Delta = \alpha \frac{(\hat{\theta}_1 - \theta_{**})^2}{\hat{\theta}_1^2} = \frac{\left(\frac{w_2}{\Sigma}\right)^2}{u} (1 - \Sigma)^2 \left(1 - \frac{\hat{\theta}_2}{\hat{\theta}_1}\right)^2.$$

We obtain

$$\frac{\Delta}{E(L_{WB}(\theta_{**}, \theta))} = (1 - \Sigma)^2 \left(\frac{w_2}{\Sigma}\right)^2 Z^2,$$

where

$$Z^2 = \frac{\left(1 - \frac{\hat{\theta}_2}{\hat{\theta}_1}\right)^2}{u(1 - w_1) - \left(\frac{w_2}{\Sigma}\right)^2}.$$

For given n , $\lambda_{w_1, w_2} = \frac{w_1 s^2}{w_1 s^2 + w_2 \bar{X}^2}$ and weights (w_1, w_2, w_3) , Δ increases with the difference between Bayesian $\hat{\theta}_2$ and non-Bayesian $\hat{\theta}_1$ estimates of θ .

If, for example, $w_2 = \Sigma = \frac{1}{2}$ and $Z = 1$ (that is $w_1 = \frac{1}{2n}$ and $w_3 = \frac{n-1}{2n}$), $\frac{\Delta}{E(L_{WB}(\theta_{**}, \theta))} = 0.25$. That is, under these conditions, expected loss is inflated by 25% using $\hat{\theta}_1$ rather than the optimal estimate θ_{**} . Similar conclusions are available for $\hat{\theta}_2$.

4 The exponential model with an informative prior for θ

Considering again the exponential model, we assume now an informative prior for θ ,

$$\pi(\theta) \sim \frac{1}{\theta^{\nu+1}} e^{-\frac{a}{\theta}}$$

where $\nu, a > 0$ (in fact, $\pi(\theta) = \frac{a^\nu}{\Gamma(\nu)} \frac{1}{\theta^{\nu+1}} e^{-\frac{a}{\theta}}$). Then

$$\pi(\theta|x_1, \dots, x_n) = \frac{(n\bar{x} + a)^{n+\nu}}{\Gamma(n+\nu)} \frac{1}{\theta^{n+\nu+1}} e^{-\frac{n\bar{x}+a}{\theta}}$$

and

$$\begin{aligned}\bar{\theta} &= E(\theta|x_1, \dots, x_n) = \frac{n\bar{x} + a}{n + \nu - 1}, \\ E(\theta^2|x_1, \dots, x_n) &= \frac{(n\bar{x} + a)^2}{(n + \nu - 1)(n + \nu - 2)}, \\ v &= \frac{(n\bar{x} + a)^2}{(n + \nu - 1)^2 (n + \nu - 2)}.\end{aligned}$$

Moreover, in this case,

$$\hat{\theta}_2 = \frac{E(\theta^2)}{\bar{\theta}} = \frac{n + \nu - 1}{n + \nu - 2} \bar{\theta}$$

and

$$\theta_{**} = \frac{w_2 \bar{X} \hat{\theta}_1 + w_3 \bar{\theta} \hat{\theta}_2}{w_2 \bar{X} + w_3 \bar{\theta}}.$$

Obviously,

$$E(L_{WB}(\theta_{**}, \theta)) = 1 - \left(w_1 + \frac{w_2 \bar{X} + w_3 \bar{\theta}}{\theta_{**}} \right).$$

We note that the posterior mean, could be written as

$$\bar{\theta} = \frac{n}{n + \nu - 1} \bar{X} + \left(1 - \frac{n}{n + \nu - 1} \right) \theta_0$$

where $\theta_0 = \frac{a}{\nu-1}$ is the prior mean, so

$$\bar{\theta} = c \bar{X} + (1 - c) \theta_0,$$

where $c = \frac{n}{n + \nu - 1} \in (0, 1)$.

If $w_2 = w_3$ and $\theta_0 > \bar{X}$ (leading to $\bar{\theta} > \bar{X}$), then $\hat{\theta}_2$ has a greater weight in expression of θ_{**} than $\hat{\theta}_1$. The opposite is true for $\theta_0 < \bar{X}$.

5 Conclusions

The three-parts weighted balanced loss function reflects goodness-of-fit and precision of estimation. Traditional loss functions such as the sum of weighted squared residuals and mean-squared error of estimation as well as the weighted balanced loss function introduced by Rodrigues and Zellner [7] can be obtained from the three-parts (weighted) balanced loss function introduced in Ciumara et al. [3], taking certain values for weights.

For example, if we analyze a three-parts weighted balanced loss function of the type presented here,

$$L_{WB}(\hat{\theta}, \theta) = w_1 \frac{2s^2}{\hat{\theta}^2} + w_2 \frac{2(\hat{\theta} - \bar{X})^2}{\hat{\theta}^2} + w_3 \frac{(\hat{\theta} - \theta)^2}{\hat{\theta}^2},$$

we get

$$\theta_{**} = \frac{2w_2\bar{X}\hat{\theta}_1 + w_3\bar{\theta}\hat{\theta}_2}{2w_2\bar{X} + w_3\bar{\theta}}$$

and

$$E(L_{WB}(\theta_{**}, \theta)) = 2w_2 + w_3 - \frac{2w_2\bar{X} + w_3\bar{\theta}}{\theta_{**}}.$$

For $w_1 = w_2 = \frac{w}{2}$, $w_3 = 1 - w$, where $w \in [0, 1]$, we get exactly the weighted balanced loss function proposed by Rodrigues and Zellner [7].

The use of a weighted-balanced loss function creates a balance between Bayesian and non-Bayesian procedures, that is, it leads to an estimate that is a linear combination of a non-Bayesian and a Bayesian estimate.

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