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On the uniform exponential stability of linear skew-product three-parameter semiflows

by
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Abstract

The asymptotic behavior of the solutions of variational differential systems is analyzed in terms of "Perron-type" theorems. We approach the theory from a uniform point of view.

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1 Introduction and Preliminaries

In the last years, there has been an increasing interest in the study of the asymptotic behavior of linear skew-product (semi)flows, in Banach spaces. Important results were obtained by Y. Latushkin , S. Montgomery-Smith and T. Randolph [5], Y. Latushkin and R. Schnaubelt [6], Y. Latushkin, A.M. Stepin [7], V.A. Pliss, G.R. Sell [13], R.J. Sacker, G.R. Sell [15], C. Chicone, Y. Latushkin [2]. The more general case of skew-evolution semiflows was also approached by C. Stoica and M. Megan [16].

The purpose of the present paper is to extend a stability result due to O. Perron [12] for the case of linear skew-product three-parameter semiflows. Other significant results concerning the extension of "Perron-type" problems for linear differential equations in the general framework of infinite-dimensional Banach spaces were due to J. L. Daleckij, M. G. Krein [3], J. L. Massera, J.J Schäffer [8, 9], N. van Minh, F. Räbiger, R. Schnaubelt [10] and P. Preda [14].

Let us consider (Θ, d) a metric space, X a Banach space, $\mathcal{B}(X)$ the space of all bounded operators acting on X and $\Delta = \{(t, t_0) \in \mathbb{R}^2 : t \geq t_0 \geq 0\}$. We denote the norm of vectors on X and operators on $\mathcal{B}(X)$ by $||\cdot||$.

Definition 1.1. *A two-parameter (nonlinear) semiflow $\sigma : \Theta \times \mathbb{R}_+ \to \Theta$ is defined by the properties:*

i) $\sigma(\theta, 0) = \theta$, *for all* $\theta \in \Theta$;

ii) $\sigma(\theta, t+s) = \sigma(\sigma(\theta, s), t)$; *for all* $\theta \in \Theta$ *and* $t, s \in \mathbb{R}_+$

If in addition $(\theta, t) \to \sigma(\theta, t)$ is continuous, then σ is called a *continuous* two-parameter (nonlinear) semiflow on Θ .

Remark 1.1. *If the above properties hold for any* $t, s \in \mathbb{R}$ *then* σ *is said to be a (nonlinear) two-parameter flow on* Θ.

Definition 1.2. *A family* $\{T(t)\}_{t\geq 0}$ *of linear and bounded operators acting on* X, *is said to be a* C_0*-semigroup on* X *if the following conditions hold:*

i) $T(0) = I$;

ii) $T(t+s) = T(t)T(s)$, *for all* $t, s \geq 0$;

iii) there exists $\lim\limits_{t\to 0_+} T(t)x = x$, *for all* $x \in X$

If the second property holds for any $t, s \in \mathbb{R}$ then $\{T(t)\}_{t\in\mathbb{R}}$ is called a C_0-group.

Remark 1.2. *It is well-known the connection between (nonlinear) (semi)flows, first order differential operators, and (linear) (semi)groups. For instance, consider a continuously differentiable vector field* $F : \mathbb{R}^n \to \mathbb{R}^n$ *with* $\sup_{\theta\in\mathbb{R}^n} ||DF(\theta)|| < \infty$, *for the derivative* $DF(\theta)$ *of* F *and* $\theta \in \mathbb{R}^n$. *Take the first order differential operator on*

$$X := C_0(\mathbb{R}^n) = \{f : \mathbb{R}^n \to \mathbb{R}^n : \text{ f is continuous vanishing at infinity }\}$$

corresponding to the vector field F,

$$Af(\theta) = \langle gradf(\theta), F(\theta)\rangle = \sum_{i=1}^{n} F_i(\theta)\frac{\partial f}{\partial \theta_i}(\theta),$$

for $f \in C_c^1(\mathbb{R}^n) = \{f : \mathbb{R}^n \to \mathbb{R}^n :$ *f continuously differentiable, with compact support* $\}$, *and* $\theta \in \mathbb{R}^n$. *It can be easily shown that* A *is dissipative. Since* F *is globally Lipschitz it follows that there exists a continuous two-parameter flow* $\sigma : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ *which solves the differential equation*

$$\frac{\partial}{\partial t}\sigma(\theta, t) = F(\sigma(\theta, t)), \text{ for all } t \in \mathbb{R} \text{ and } \theta \in \mathbb{R}^n \text{ (see [1]).}$$

To such a flow we associate a one-parameter group of linear operators on $C_0(\mathbb{R}^n)$ *given by*

$$(T(t)f)(\theta) := f(\sigma(\theta, t)), \text{ for } \theta \in \mathbb{R}^n, \ t \in \mathbb{R},$$

the so-called group induced by the flow σ. *It can be proved that the generator of the above group is the closure of the first order differential operator* A. *For details we refer the reader to [4].*

The general relation between (nonlinear) two-parameter semiflows and linear semigroups is given in the example below.

Example 1.1. *Let Θ be a compact metric space and take $X = \mathcal{C}(\Theta)$, where*

$$\mathcal{C}(\Theta) = \{f : \Theta \to \mathbb{C} : f \text{ continuous on } \Theta\}$$

i) The two-parameter (nonlinear) semiflow σ is continuous if and only if it induces a strongly continuous semigroup $\{T(t)\}_{t\geq 0}$ on X by the formula:

$$(T(t)f)(\theta) := f(\sigma(\theta, t)), \text{ for } \theta \in \Theta, \ t \geq 0, \ f \in X \qquad (\diamond)$$

ii) The generator $(A, D(A))$ of $\{T(t)\}_{t\geq 0}$ is a derivation.

iii) Every strongly continuous semigroup $\{T(t)\}_{t\geq 0}$ on X that consists of algebra homomorphisms originates, via ($\diamond$), from a continuous two-parameter (nonlinear) semiflow on Θ. (see[11, B-II, Thm. 3.4])

Taking into account the relation between two-parameter (nonlinear) semiflows and strongly continuous semigroups (see the example above) we introduce a three-parameter (nonlinear) semiflow to analyze the case of two-parameter evolution families.

Definition 1.3. *A three-parameter(nonlinear) semiflow $\sigma : \Theta \times \Delta \to \Theta$ is defined by the properties:*

i) $\sigma(\theta, t, t) = \theta$ for all $t \in \Delta$ and all $\theta \in \Theta$

ii) $\sigma(\sigma(\theta, s, t_0), t, s) = \sigma(\theta, t, t_0)$ for all $t, s, t_0 \in \Delta$ and all $\theta \in \Theta$.

If in addition $(\theta, t, t_0) \mapsto \sigma(\theta, t, t_0)$ is continuous then σ is called a *continuous* three-parameter (nonlinear) semiflow on Θ.

Definition 1.4. *The pair $\pi = (\Phi, \sigma)$ is said to be a linear skew-product three-parameter semiflow on X if $\Phi : \Theta \times \Delta \to \mathcal{B}(X)$ satisfies the following properties:*

i) $\Phi(\theta, t, t) = I$ for all $t \in \Delta$ and all $\theta \in \Theta$, where I represents the identity operator on X;

ii) $\Phi(\sigma(\theta, s, t_0), t, s)\Phi(\theta, s, t_0) = \Phi(\theta, t, t_0)$, for all $t, s, t_0 \in \Delta$ and all $\theta \in \Theta$.

iii) $t \mapsto \Phi(\theta, t, t_0)x : [t_0, \infty) \to X$ is continuous;
$\tau \mapsto \Phi(\theta, t, \tau)x : [0, t] \to X$ is continuous;

iv) there exist $M, \omega \in \mathbb{R}$, $M \geq 1$ such that

$$||\Phi(\theta, t, t_0)|| \leq Me^{\omega(t-t_0)}, \text{ for all } t \geq t_0 \geq 0, \text{ and } \theta \in \Theta$$

Example 1.2. *Let σ be a continuous three-parameter (nonlinear) semiflow on Θ, $A : \Theta \to \mathcal{B}(X)$ a continuous map and f a locally integrable function on X.*

It is easy to see that the solution of the homogeneous Cauchy problem:

$$\begin{cases} \dot{x}(t) = A(\sigma(\theta, t, t_0))x(t), \quad t \geq t_0 \geq 0 \\ x(t_0) = x_0 \end{cases}$$

verifies the integral equation

$$x(t) = x_0 + \int_{t_0}^{t} A(\sigma(\theta, \tau, t_0))x(\tau)d\tau, \quad (1.2.1)$$

and that of the inhomogeneous Cauchy problem:

$$\begin{cases} \dot{x}(t) = A(\sigma(\theta, t, t_0))x(t) + f(t), \quad t \geq t_0 \geq 0 \\ x(t_0) = x_0 \end{cases}$$

verifies the integral equation

$$x(t) = \Phi(\theta, t, t_0)x_0 + \int_{t_0}^{t} A(\sigma(\theta, \tau, t_0), t, \tau)x(\tau)d\tau + \int_{t_0}^{t} f(\tau)d\tau \quad (1.2.2)$$

due to the fact that in both cases, the solution of the variational Cauchy problem is an absolutely continuous function.

With a similar argument as in the proof of the existence and uniqueness theorems for the non-autonomous systems (see for instance J.L. Daleckij, M.G. Krein [3] and J.L. Massera, J.J. Schäffer [9]), one can show that the solution of the variational homogeneous equation (1.2.1) is $\Phi(\theta, t, t_0)x_0 = x(t)$, where $\Phi(\theta, t, t_0)$ is a linear skew-product three-parameter semiflow, and that (1.2.2) has the solution

$$x(t) = \Phi(\theta, t, t_0)x_0 + \int_{t_0}^{t} \Phi(\sigma(\theta, \tau, t_0), t, \tau)f(\tau)d\tau.$$

Example 1.3. *Let $\Theta = \mathbb{R}$. The mapping*

$$\sigma : \Theta \times \Delta \to \Theta, \ \sigma(\theta, t, t_0) = \theta + t - t_0$$

is a three-parameter semiflow on $\mathbb{R}$.

Consider now the continuous and bounded mapping $a : \mathbb{R} \to \mathbb{R}$ and the variational equation

$$\begin{cases} \dot{x}(t) = a(\theta + t - t_0)x(t) \\ x(t_0) = 1 \end{cases}$$

Then

$$x(t) = e^{\int_{t_0}^{t} a(\theta+s-t_0)ds}.$$

By taking

$$\Phi(\theta, t, t_0) = e^{\int_{t_0}^{t} a(\theta+s-t_0)ds}$$

we have that $\Phi(\theta, t, t_0)$ *is a linear skew-product three-parameter semiflow on* Θ.

Definition 1.5. *A family of linear and bounded operators* $\{U(t, t_0)\}_{t \geq t_0 \geq 0}$ *is said to be a two-parameter evolution family if it satisfies the following conditions:*

i) $U(t, t) = I$, *for all* $t \geq 0$*;*

ii) $U(t, s)U(s, t_0) = U(t, t_0)$, *for all* $t \geq s \geq t_0 \geq 0$*;*

iii) $U(\cdot, t_0)x$ *is continuous on* $[t_0, \infty)$, *for all* $t_0 \geq 0,\ x \in X$*;*
$U(t, \cdot)x$ *is continuous on* $[0, t]$, *for all* $t \geq 0,\ x \in X$*;*

iv) there exist $M \geq 1$ *and* $\omega \in \mathbb{R}$ *such that*

$$||U(t, t_0)|| \leq Me^{\omega(t-t_0)},\ \textit{for all } t \geq t_0 \geq 0.$$

Example 1.4. *If* $\{U(t, t_0)\}_{t \geq t_0 \geq 0}$ *is a two-parameter evolution family on* X *and* σ *is a three-parameter semiflow on* Θ, *then* $\pi = (\Phi, \sigma)$ *is a linear skew-product three-parameter semiflow, where*

$$\Phi(\theta, t, t_0) = U(t, t_0).$$

Thus, we can consider that evolution families are particular cases of linear skew-product three-parameter semiflows.

Conversely, considering $\Theta = \mathbb{R}_+,\ \sigma : \mathbb{R}_+ \times \Delta \to \mathbb{R}_+,\ \sigma(\theta, t, t_0) = \theta$, *and* $\pi = (\Phi, \sigma)$ *a linear skew-product three-parameter semiflow on* X, *we have that* $\{U(t, t_0)\}_{t \geq t_0 \geq 0},\ U(t, t_0) = \Phi(0, t, t_0)$ *is an evolution family on* X.

Example 1.5. *Let* $\Theta = \mathbb{R}_+$ *and* σ *be the linear three-parameter semiflow given in Example 1.3. If* $\{U(t, t_0)\}_{t \geq t_0 \geq 0}$ *is a two-parameter evolution family on* X, *then* $\pi = (\Phi, \sigma)$ *is a linear skew-product three-parameter semiflow, where*

$$\Phi(\theta, t, t_0) = U(\theta + t - t_0, \theta).$$

Definition 1.6. *Let* $\pi = (\Phi, \sigma)$ *be a linear skew-product three-parameter semiflow.* $\pi = (\Phi, \sigma)$ *is said to be uniformly exponentially stable if there exist* $N \geq 1$ *and* $\nu > 0$ *such that:*

$$||\Phi(\theta, t, t_0)x|| \leq Ne^{-\nu(t-t_0)}||x||,\ \forall\ x \in X,\ \forall\ \theta \in \Theta,\ \forall\ t \geq t_0 \geq 0.$$

2 The main result

Let us consider

$$\mathcal{C}(\mathbb{R}_+, X) = \{f : \mathbb{R}_+ \to X : \ f \text{ is continuous and bounded on } \mathbb{R}_+\}$$

which is a Banach space endowed with the norm $|||f||| = \sup_{t\geq 0} ||f(t)||$ and

$$\mathcal{C}_b(\mathbb{R}_+, X) = \{f : \mathbb{R}_+ \to X : \ f \text{ is bounded on } \mathbb{R}_+\}.$$

For $f \in \mathcal{C}(\mathbb{R}_+, X)$ we consider the mapping

$$\tilde{x}_f : \Theta \times \mathbb{R}_+ \to \mathcal{C}(\mathbb{R}_+, X), \ \tilde{x}_f(\theta, t_0) = \bar{x}_f(\theta, \cdot, t_0),$$

where

$$\bar{x}_f(\theta, t, t_0) = \begin{cases} 0, \ t < t_0 \\ x_f(\theta, t, t_0), \ t \geq t_0 \end{cases}$$

and

$$x_f(\theta, t, t_0) = \int_{t_0}^{t} \Phi(\sigma(\theta, \tau, t_0), t, \tau) f(\tau) d\tau \qquad (2.1.1).$$

Definition 2.1. *Let $\pi = (\Phi, \sigma)$ be a linear skew-product three-parameter semiflow. $\pi = (\Phi, \sigma)$ satisfies the Perron condition if for all $f \in \mathcal{C}(\mathbb{R}_+, X)$, the mapping $\tilde{x}_f \in \mathcal{C}_b(\Theta \times \mathbb{R}_+, \mathcal{C}(\mathbb{R}_+, X))$. (where $\mathcal{C}_b(\Theta \times \mathbb{R}_+, \mathcal{C}(\mathbb{R}_+, X))$ represents the set of all bounded $\mathcal{C}(\mathbb{R}_+, X)$-valued applications on $\Theta \times \mathbb{R}_+$)*

The following theorem is essential in the proof of the central result of this paper.

Theorem 2.1. *Let $\pi = (\Phi, \sigma)$ be a linear skew-product three-parameter semiflow. If $\pi = (\Phi, \sigma)$ satisfies the Perron condition, then there exists a constant $k > 0$ such that*

$$|||\tilde{x}_f||| \leq k|||f|||, \ \textit{for all } f \in \mathcal{C}(\mathbb{R}_+, X).$$

Proof: Define

$$\mathcal{U} : \mathcal{C}(\mathbb{R}_+, X) \to \mathcal{C}_b(\Theta \times \mathbb{R}_+, \mathcal{C}(\mathbb{R}_+, X)), \ \mathcal{U}f = \tilde{x}_f,$$

where $\tilde{x}_f$ is defined as above. We note that $\mathcal{U}$ is a linear operator. In order to show that $\mathcal{U}$ is also bounded, we note that

$$||x_{f_n}(\theta, t, t_0) - x_f(\theta, t, t_0)|| \leq \int_{t_0}^{t} ||\Phi(\sigma(\theta, \tau, t_0), t, \tau)(f_n(\tau) - f(\tau))||d\tau$$

$$\leq \int_{t_0}^{t} ||\Phi(\sigma(\theta, \tau, t_0), t, \tau)||\ ||f_n(\tau) - f(\tau)|| d\tau$$

$$\leq M e^{\omega(t-t_0)}(t - t_0)|||f_n - f|||.$$

Since

$$f_n \to f \text{ and } \mathcal{U} f_n \to g, \text{ with respect to the norm } ||| \cdot |||$$

it follows that

$$x_{f_n}(\theta, t, t_0) \to x_f(\theta, t, t_0), \text{ for } n \to \infty \text{ and for all } t \geq t_0, \theta \in \Theta,$$

thus $\tilde{x}_f = g$. From the Closed Graph Theorem we obtain that there exists $k > 0$ such that

$$|||\tilde{x}_f||| \leq k|||f|||, \text{ for all } f \in \mathcal{C}(\mathbb{R}_+, X).$$

□

In order to establish sufficient conditions for the uniform exponential stability of the linear skew-product three-parameter semiflows, we will use a result due to J. L. Massera and J. J. Schäffer [8]:

Lemma 2.1. *Take* $f, g : \mathbb{R}_+ \to \mathbb{R}_+,$ g *continuous, such that*

i) $f(t) \leq g(t - t_0) f(t_0)$, *for all* $t \geq t_0 \geq 0$;

ii) $\inf_{t \geq 0} g(t) < 1$.

Then there exist $N, \nu > 0$ *such that*

$$f(t) \leq N e^{-\nu(t-t_0)} f(t_0), \textit{ for all } t \geq t_0 \geq 0.$$

Now we can state the main result of this paper:

Theorem 2.2. *Let* $\pi = (\Phi, \sigma)$ *be a linear skew-product three-parameter semiflow.* $\pi = (\Phi, \sigma)$ *is uniformly exponentially stable if and only if* $\pi = (\Phi, \sigma)$ *satisfies the Perron condition.*

Proof: The *necessity* is immediate. For the *sufficiency* part let us consider $t_0 > 0, \delta > 0$ and $\theta \in \Theta$, as well as

$$f(t) = \begin{cases} 0, \text{ if } 0 \leq t \leq t_0 - \delta \\ \frac{1}{\delta}(t - t_0 + \delta)x, \text{ if } t_0 - \delta < t < t_0 \\ e^{-\omega(t-t_0)}\Phi(\theta, t, t_0)x, \text{ if } t \geq t_0, \end{cases}$$

From

$$||\Phi(\theta, t, t_0)|| \leq M e^{\omega(t-t_0)}, \text{ for all } t \geq t_0 \geq 0, \text{ and } \theta \in \Theta$$

it follows that $f \in \mathcal{C}(\mathbb{R}_+, X)$ and $|||f||| \leq M||x||$. Thus, for $t \geq t_0$ we have that

$$x_f(\theta, t, t_0) = \int_{t_0}^{t} \Phi(\sigma(\theta, \tau, t_0), t, \tau) f(\tau) d\tau$$

$$= \int_{t_0}^{t} e^{-\omega(\tau - t_0)} \Phi(\sigma(\theta, \tau, t_0), t, \tau) \Phi(\theta, \tau, t_0) x d\tau$$

$$= \int_{t_0}^{t} e^{-\omega(\tau - t_0)} \Phi(\theta, t, t_0) x d\tau$$

$$= \frac{1}{\omega}\Big(1 - e^{-\omega(t - t_0)}\Big) \Phi(\theta, t, t_0) x.$$

Using the result from Theorem 2.1 it follows that

$$||x_f(\theta, t, t_0)|| \leq k|||f||| \leq kM||x||, \text{ for all } t \geq t_0 \geq 0, \text{ and } \theta \in \Theta.$$

But from

$$||\frac{1}{\omega}\Big(1 - e^{-\omega(t - t_0)}\Big) \Phi(\theta, t, t_0) x|| \leq kM||x||,$$

it results that

$$||\Phi(\theta, t, t_0)x - e^{-\omega(t - t_0)} \Phi(\theta, t, t_0) x|| \leq kM\omega||x||.$$

Then

$$\begin{aligned} ||\Phi(\theta, t, t_0)x|| &\leq ||\Phi(\theta, t, t_0)x - e^{-\omega(t - t_0)} \Phi(\theta, t, t_0) x|| \\ &+ ||e^{-\omega(t - t_0)} \Phi(\theta, t, t_0) x|| \\ &\leq kM\omega||x|| + M||x|| \\ &= (1 + k\omega)M||x||, \end{aligned}$$

for all $t \geq t_0 \geq 0,\ x \in X$ and $\theta \in \Theta$. Thus we have obtained that

$$||\Phi(\theta, t, t_0)x|| \leq (1 + k\omega)M||x||. \qquad (2.2.1)$$

Define now

$$g(t) = \begin{cases} 0, \text{ if } 0 \leq t \leq t_0 - \delta \\ \frac{1}{\delta}(t - t_0 + \delta)x, \text{ if } t_0 - \delta < t < t_0 \\ \Phi(\theta, t, t_0)x, \text{ if } t \geq t_0, \end{cases}$$

We note that $g \in \mathcal{C}(\mathbb{R}_+, X),\ |||g||| \leq (1 + k\omega)M||x||$ and

$$x_g(\theta, t, t_0) = \int_{t_0}^{t} \Phi(\sigma(\theta, \tau, t_0), t, \tau) g(\tau) d\tau$$

$$= \int_{t_0}^{t} \Phi(\sigma(\theta, \tau, t_0), t, \tau)\Phi(\theta, \tau, t_0)x d\tau$$

$$= \int_{t_0}^{t} \Phi(\theta, t, t_0)x d\tau$$

$$= (t - t_0)\Phi(\theta, t, t_0)x.$$

From Theorem 2.1 it follows that

$$||x_g(\theta, t, t_0)|| \leq k|||g||| \leq k(1 + k\omega)M||x||, \text{ for all } t \geq t_0 \geq 0, \ x \in X \text{ and } \theta \in \Theta$$

and therefore

$$(t - t_0)||\Phi(\theta, t, t_0)x|| \leq k(1 + k\omega)M||x||. \qquad (2.2.2)$$

Using the relations (2.2.1) and (2.2.2) we obtain that

$$(1 + t - t_0)||\Phi(\theta, t, t_0)x|| \leq L||x||,$$

where $L = M(1 + k\omega)(1 + k)$. Hence

$$||\Phi(\theta, t, t_0)x|| \leq \frac{L}{1 + t - t_0}||x||, \text{ for all } t \geq t_0 \geq 0, \ x \in X \text{ and } \theta \in \Theta.$$

But

$$||\Phi(\theta, t, t_0)x|| \leq ||\Phi(\sigma(\theta, s, t_0), t, s)|| \ ||\Phi(\theta, s, t_0)x|| \leq \frac{L}{1 + t - s}||\Phi(\theta, s, t_0)x||,$$

for all $t \geq s \geq t_0, \ x \in X, \ \theta \in \Theta$ and using the result from Lemma 2.1 we obtain that there exist $N, \nu > 0$ such that

$$||\Phi(\theta, t, t_0)x|| \leq Ne^{\nu(t - t_0)}||x||, \text{ for all } t \geq t_0 \geq 0, \ x \in X \text{ and } \theta \in \Theta,$$

hence $\pi = (\Phi, \sigma)$ is uniformly exponentially stable.

$\square$

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